Polar flocks with discretized directions: the active clock model approaching the Vicsek model

Matthieu Mangeat[1], Swarnajit Chatterjee[1], and Heiko Rieger[1]

[1]Center for Biophysics & Department for Theoretical Physics, Saarland University, Saarbrücken, Germany

Introduction

Collective motions are widely observed in nature (bird flocks, fish schools, bacteria swarms) and also studied in various artificial systems (liquid droplets, rolling colloids, vibrated polar disks). They consist of a large number of active particles which consume energy to self-propel and interact via alignment and/or repulsion. For large densities and low noise spontaneous synchronized motion of large clusters of particles, denoted as flocks, emerges. This flocking transition is an out-of-equilibrium phenomenon, leading to several theoretical models.

The first theoretical model displaying a flocking transition was the Vicsek model (VM) [1, 2]. The active particles perform a ballistic motion with constant velocity and align with the local average particle direction with the rule $\theta_i(t+dt) = \langle\theta(t)\rangle_r + \eta\xi_i(t)$. This alignment is similar to a ferromagnetic interaction where η plays the role of the temperature. A collective motion emerges at low temperatures and high densities due to a spontaneous breaking of the continuous symmetry. A continuum theory proposed by Toner and Tu [3] relates the Vicsek model to the XY model universality class.

Recently, Solon *et al.* [4] studied a discrete model - called active Ising model (AIM) - on a two-dimensional lattice with active particles hopping with bias to the left or to the right and aligning locally like ferromagnetic Ising spins. The flocking transition of this model is very similar to the VM and allows various analytical results. We have studied a generalization for the q-state active Potts model (APM) [5, 6] and the q-state active clock model (ACM) [7] in which self-propelled particles have q internal states (directions of motion) and align locally like Potts and clock spins, respectively. In this poster we present the results obtained for the q-state ACM.

The microscopic model

- N particles in a off-lattice $L_x \times L_y$ rectangular domain. Average density: $\rho_0 = N/L_xL_y$.
- ith particle is characterized by its position $\mathbf{x_i}$ and orientation $\theta_i \in \{0, 2\pi/q, 4\pi/q, \ldots, 2(q-1)\pi/q\}$.
- Neighborhood $\mathcal{N}_i = \{j \text{ with } |\mathbf{x_i} - \mathbf{x_j}| < 1\}$ of ith particle is constituted of ρ_i particles (no restriction).
- Local clock Hamiltonian and local magnetization in the neighborhood $\mathcal{N}_i$:

$$H_i = -\frac{J}{2\rho_i}\sum_{k\in\mathcal{N}_i}\sum_{\substack{l\in\mathcal{N}_i\\ l\neq k}}\cos(\theta_l - \theta_k), \qquad \mathbf{m_i} = \sum_{k\in\mathcal{N}_i}(\cos\theta_k, \sin\theta_k).$$

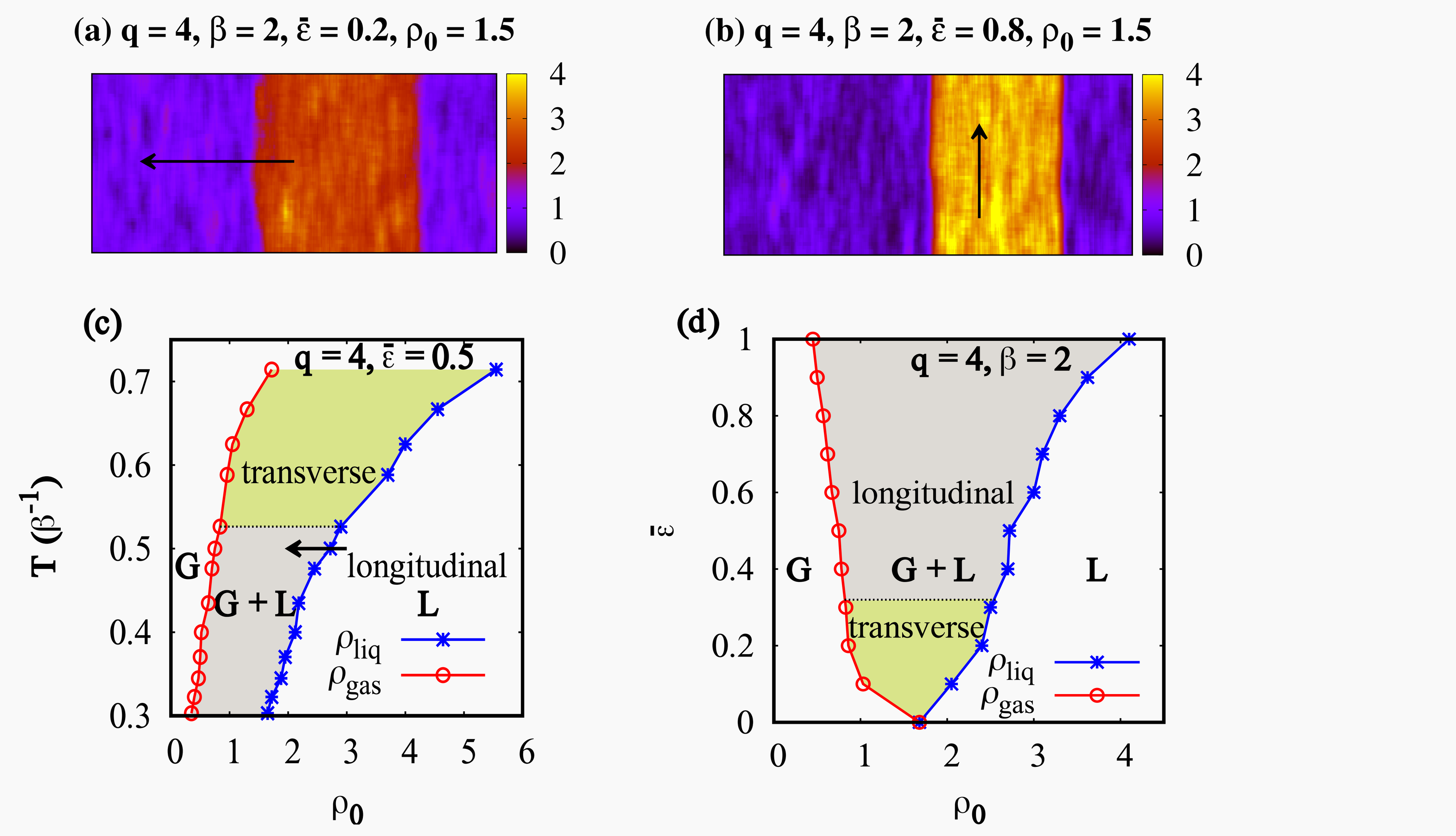
- At each time t, the ith particle in state $\theta_i = \theta$ can
  - hop in preferred direction $\phi = \theta$ with a rate: $D(1+\varepsilon)$,
  - hop in another direction $\phi \neq \theta$ with a rate: $D[1 - \varepsilon/(q-1)]$,
  - flip to an orientation $\theta' \neq \theta$, for a temperature $T = \beta^{-1}$, with a rate:

$$W_{\text{flip}}(\theta \to \theta') = \gamma\exp(-\beta\Delta H_i) = \gamma\exp\left\{\frac{\beta J}{\rho_i}\left[\mathbf{m_i}\cdot(\mathbf{e}_{\theta'} - \mathbf{e}_\theta) + 1 - \cos(\theta' - \theta)\right]\right\}.$$

- Rescaled quantities (for an infinite q limit well-defined):

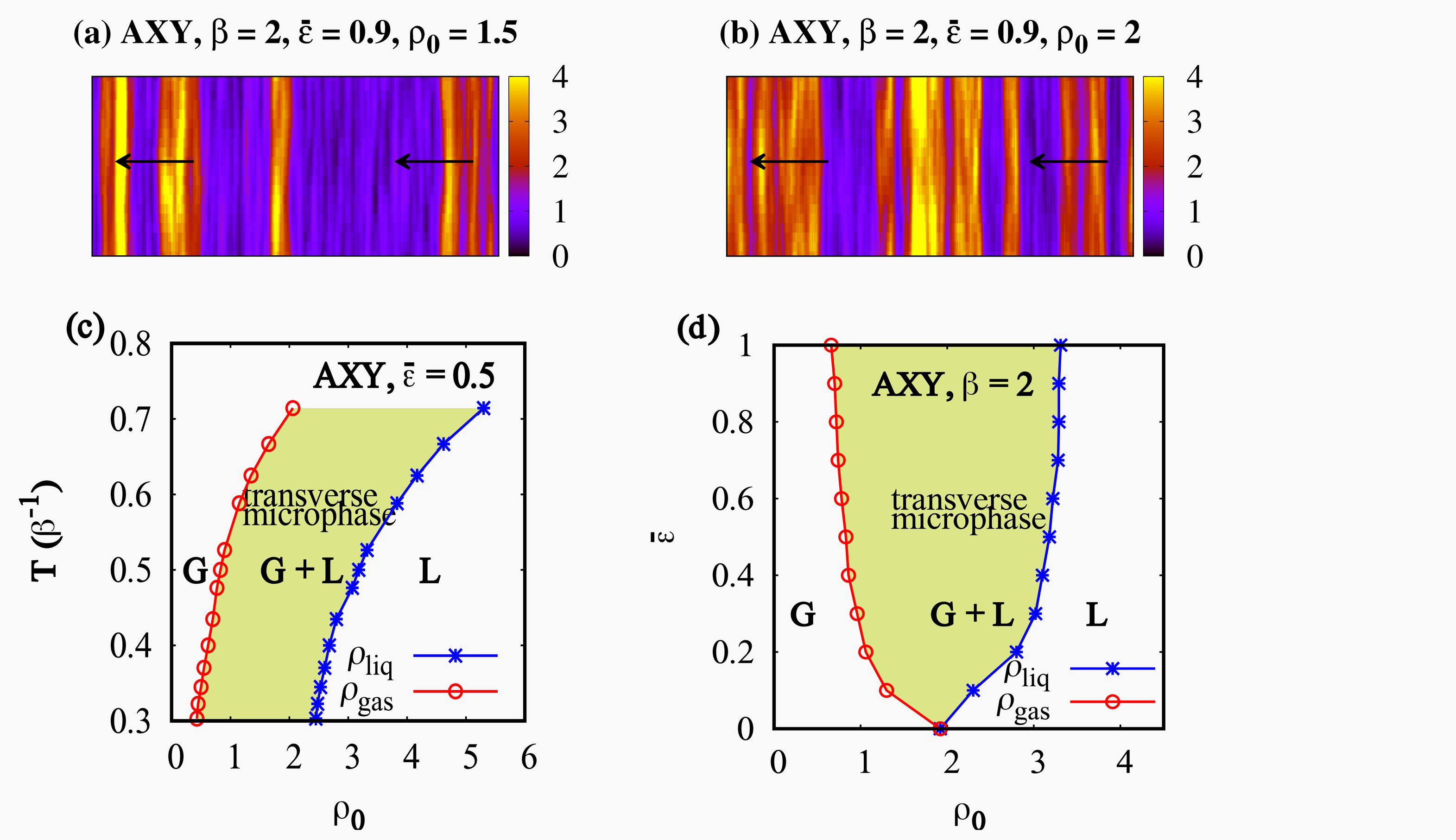
$$\overline{D} = qD \quad \text{and} \quad \overline{\varepsilon} = \frac{\varepsilon}{q-1}.$$

$q = 4$-state active clock model



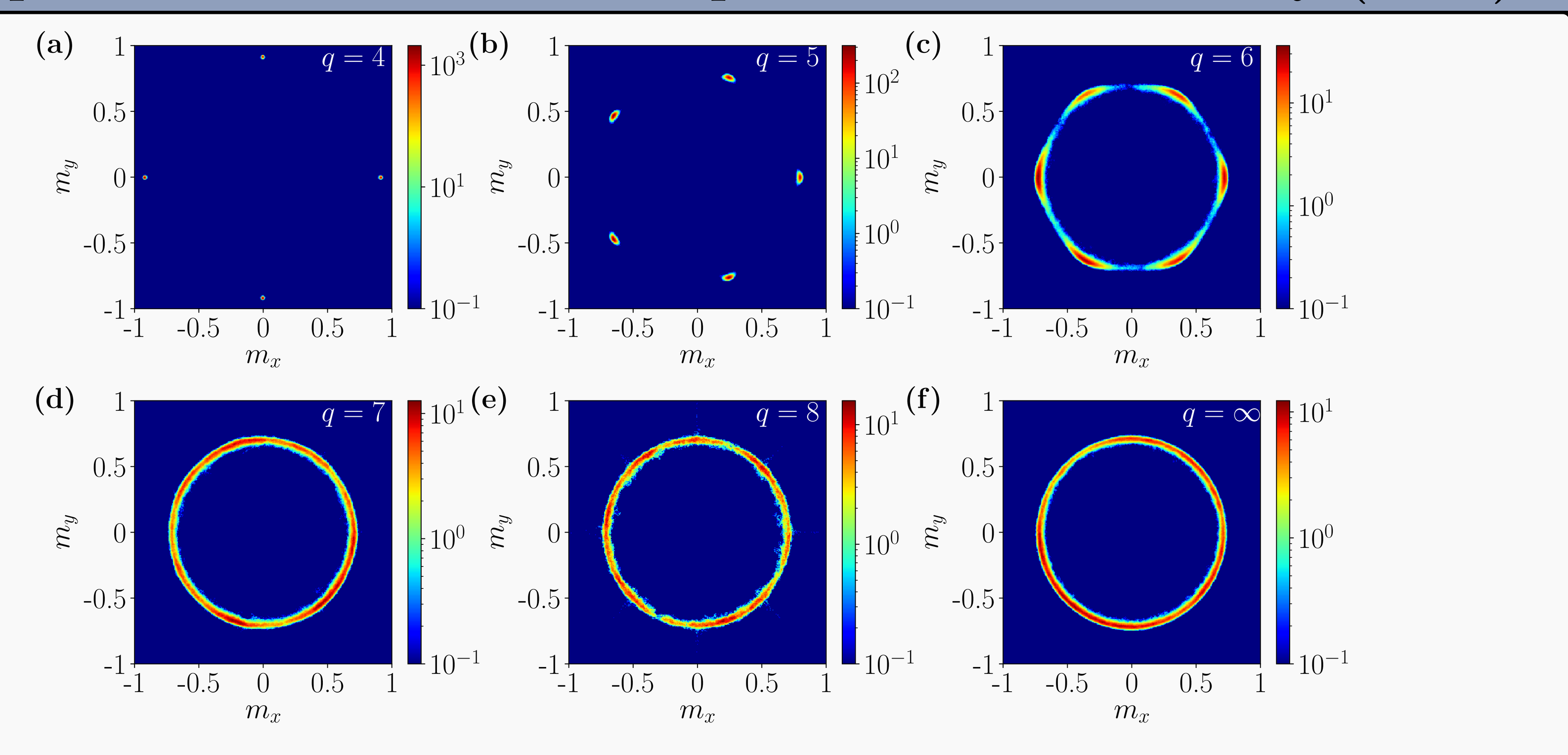
- Liquid-gas phase transition, with macrophase separation in the coexistence region (like AIM and APM).
- Presence of the reorientation transition (like APM), from transverse band (low bias ε) to longitudinal lane (high bias ε). This transition is due to the heterogeneous diffusion in longitudinal $[D_\parallel = D(1+\overline{\varepsilon})]$ and transverse $[D_\perp = D(1-\overline{\varepsilon})]$ directions, leading to $D_\perp \ll D_\parallel$ when $\overline{\varepsilon} \to 1$.

active XY model ($q = \infty$)



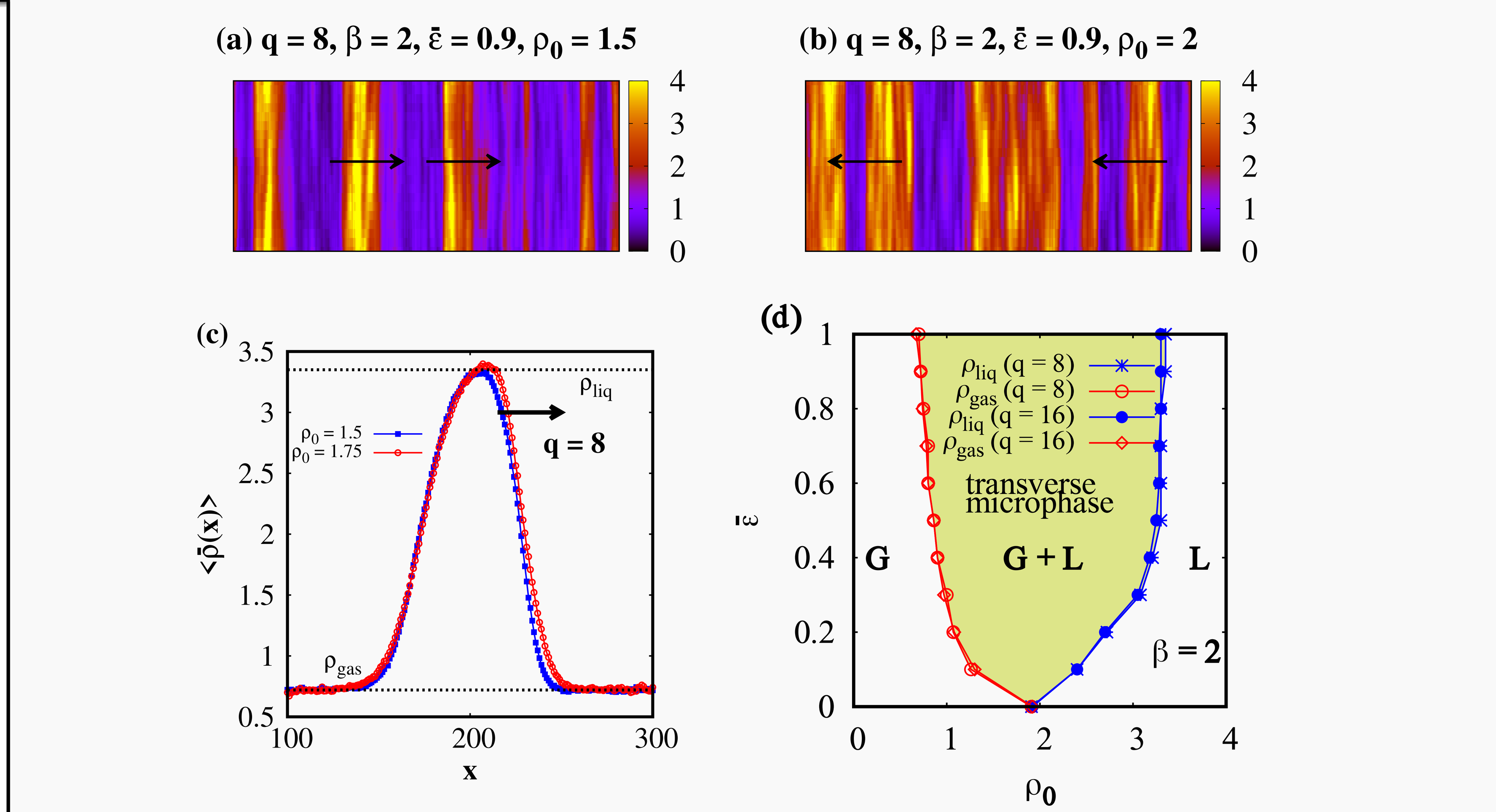
- Liquid-gas phase transition, with microphase separation in the coexistence region (like VM).
- Absence of the reorientation transition (only transverse bands are observed).

q-dependence on the ordered phase without activity ($\varepsilon = 0$)



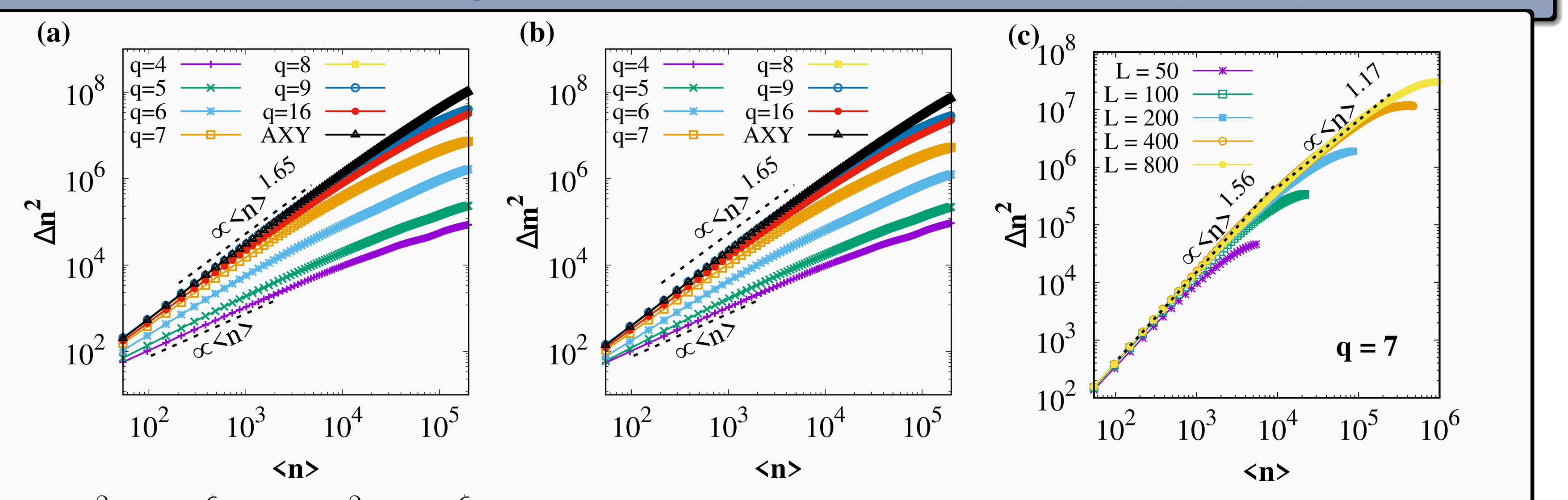
- For $q \leq 5$: "pinned" orientations (LRO), and for $q \geq 6$: "unpinned" orientations (QLRO).
- The transition QLRO/LRO may depend on the temperature, the density and the system size.

Finite q values: coexistence region and phase diagram



- Liquid-gas phase transition, with microphase separation in the coexistence region (like VM).
- Absence of the reorientation transition.

Number and magnetization fluctuations in liquid phase



- $\Delta n^2 \sim \langle n\rangle^\xi$ and $\Delta m^2 \sim \langle n\rangle^\xi$, with ξ:

q	4	5	6	7	8	16	∞
ξ	1.04	1.08	1.36	1.56	1.62	1.62	1.65

- macrophase separation in coexistence region $\Rightarrow \xi = 1$, normal fluctuations ($q \leq 5$).
- microphase separation in coexistence region $\Rightarrow \xi > 1$, giant fluctuations ($q \geq 6$).
- No asymptotic regime at large $\langle n\rangle$ observed for our system sizes ($L_x = L_y \leq 800$).

Hydrodynamic description

- Hydrodynamic equations for the density $\rho(\mathbf{x};t)$ and the magnetization $\mathbf{m}(\mathbf{x};t)$:

$$\partial_t\rho = D_0\nabla^2\rho + \frac{v}{4}\nabla\cdot(\nabla\cdot Q) - v\nabla\cdot\mathbf{m},$$

$$\partial_t\mathbf{m} = D_0\nabla^2\mathbf{m} + \frac{v}{8}\begin{pmatrix}\partial_{xx} - \partial_{yy} & 2\partial_{xy}\\ 2\partial_{xy} & -\partial_{xx} + \partial_{yy}\end{pmatrix}\mathbf{m} - \frac{v}{2}(\nabla\rho + \nabla\cdot Q) + \gamma_0\left[\beta J - 1 - \frac{r}{\rho^{2-\xi}} - \kappa\frac{\mathbf{m}^2}{\rho^2}\right]\mathbf{m}.$$

- Diffusion constant $D_0 = \overline{D}/4$, self-propulsion velocity $v = \overline{D}\overline{\varepsilon}$, ferromagnetic interaction strength $\gamma_0 = q\gamma/(q-1)$, $\kappa = (\beta J)^2(7 - 3\beta J)/8$, and nematic tensor

$$Q = \frac{\beta J}{2\rho}\begin{pmatrix}m_x^2 - m_y^2 & 2m_xm_y\\ 2m_xm_y & -m_x^2 + m_y^2\end{pmatrix}.$$

- Impossible to conclude about macrophase/microphase separation in the coexistence region with vectorial PDEs [2]. We need to introduce a noise term (future work).

Open questions - Perspectives

- Do the number fluctuations in the liquid phase have an impact on the coexistence region?
- Are the pinned/unpinned orientations in liquid phase equivalent to macrophase/microphase separation in coexistence region?
- Are the pinned/unpinned orientations equivalent to normal/giant number fluctuations in liquid phase?
- Many differences observed compared to the study made by Solon *et al.* [8], for a slightly different model (different hopping/flipping rates). From where these differences arise?
- Work on the hydrodynamic description to validate the simulation's results.

References

[1] T. Vicsek, A. Czirok, E. Ben-Jacob, I. Cohen and O. Shochet, Phys. Rev. Lett. **75**, 1226 (1995).

[2] A. P. Solon, H. Chaté and J. Tailleur, Phys. Rev. Lett. **114**, 068101 (2015).

[3] J. Toner and Y. Tu, Phys. Rev. Lett. **75**, 4326 (1995).

[4] A. P. Solon and J. Tailleur, Phys. Rev. Lett. **111**, 078101 (2013); Phys. Rev. E **92**, 042119 (2015).

[5] S. Chatterjee, M. Mangeat, R. Paul and H. Rieger, EPL **130**, 66001 (2020).

[6] M. Mangeat, S. Chatterjee, R. Paul and H. Rieger, Phys. Rev. E **102**, 042601 (2020).

[7] S. Chatterjee, M. Mangeat, and H. Rieger, EPL **138**, 41001 (2022).

[8] A. Solon, H. Chaté, J. Toner, and J. Tailleur, Phys. Rev. Lett. **128**, 208004 (2022).

Acknowledgements

This work was financially supported by the German Research Fondation (DFG) within the Collaborative Research Center SFB 1027.

mangeat@lusi.uni-sb.de