# Polar flocks with discretized directions: the active clock model approaching the Vicsek model

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Monday, March 27th

DPG Meeting - Dresden 2023 (DY 10.2)

1. **Introduction on collective motions**

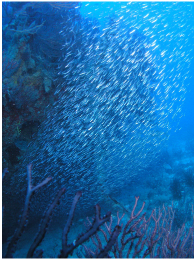
2. **Different theoretical flocking models**

3. **Active clock model**

4. **Conclusion**

## Natural and artificial collective motions

Fish schools



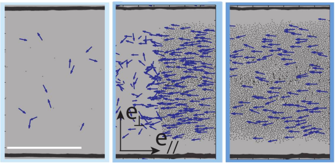
Becco et al., Physica A (2006)

Bird flocks


Ballerini et al., PNAS (2008)

bacteria swarm, human crowds, ...

Rolling colloids




Bricard et al., Nature (2013)

liquid droplets, vibrating polar disks, ...

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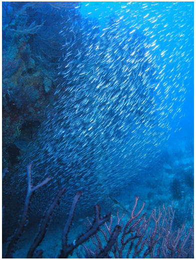

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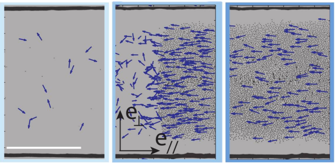


Bricard et al., Nature (2013)

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### How to create a collective motion?

- Out-of-equilibrium phenomenon: active matter system with a large number of particles:
  - consuming internal energy to self-propel,
  - interacting via alignment and/or repulsion.
- Flocking transition: spontaneous synchronized motion of large clusters (*flocks*) emerges for large densities and/or low noise.

## First theoretical model (2D): The Vicsek model (1995)

- Self-propelled particles with constant speed, aligning in the local average direction.

$$\theta_i(t+1) = \langle\theta(t)\rangle_r + \eta\xi_i(t)$$
$$\mathbf{x_i}(t+1) = \mathbf{x_i}(t) + v\mathbf{e_i}(t+1)$$

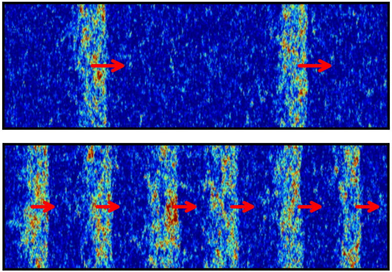
with $\mathbf{e_i}(t+1)$ in the direction of $\theta_i(t+1)$ and $\xi_i(t)$ a random number in $[-\pi,\pi]$.

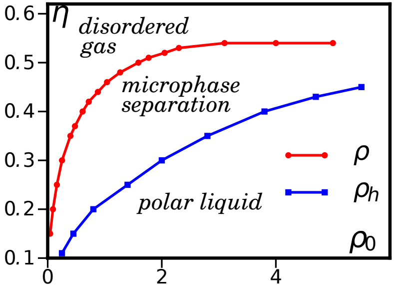
- Ferromagnetic interactions where $\eta$ plays the role of the temperature.

T. Vicsek et al., Phys. Rev. Lett. **75**, 1226 (1995)

- Hydrodynamic limit belongs to universality class of the XY model.

J. Toner and Y. Tu, Phys. Rev. Lett. **75**, 4326 (1995)

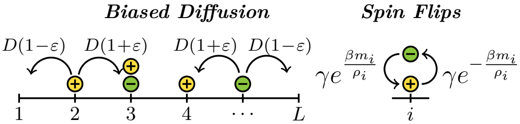




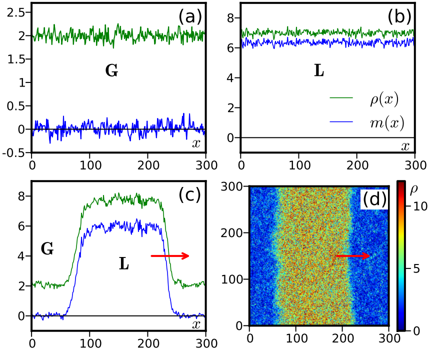

- Spontaneous breaking of the continuous symmetry.
- First-order liquid-gas phase transition, with microphase separation.

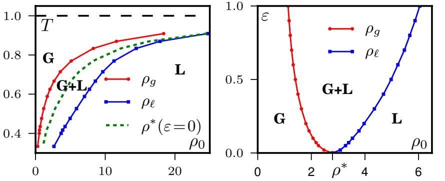
A. P. Solon et al., Phys. Rev. Lett. **114**, 068101 (2015)

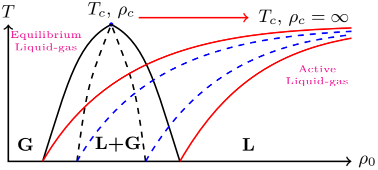
# The active Ising model (AIM)




$$W_{\text{hop}} = D(1 \pm \varepsilon)$$
$$W_{\text{flip}}(\sigma \to -\sigma) = \gamma \exp(-\beta \sigma m_i / \rho_i)$$




- The flocking transition is a first-order liquid-gas phase transition, without the supercritical region ($T_c = 1$).

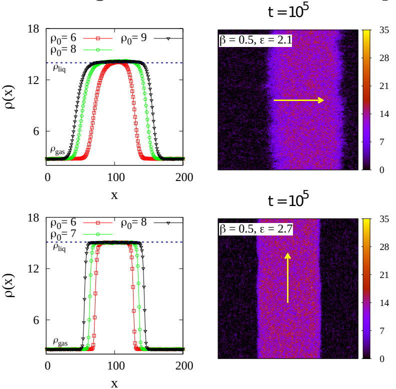
A. P. Solon and J. Tailleur, Phys. Rev. Lett. **111**, 078101 (2013)
A. P. Solon and J. Tailleur, Phys. Rev. E **92**, 042119 (2015)

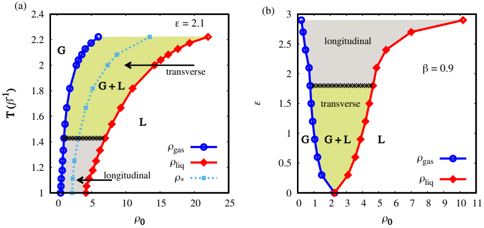
## The q-state active Potts model (APM) [$q = 4$ and $q = 6$]

- Hopping rate in preferred direction: $D(1+\varepsilon)$ and in other directions: $D\left[1-\varepsilon/(q-1)\right]$.
- Local Hamiltonian and fipping rate on site $i$:

$$H_i^{\mathrm{APM}} = -\frac{J}{2\rho_i}\sum_{k=1}^{\rho_i}\sum_{l\neq k}(q\delta_{\sigma_i^k,\sigma_i^l}-1), \qquad W_{\mathrm{flip}}(\sigma\to\sigma') \propto \exp(-\beta\Delta H_i^{\mathrm{APM}}).$$

- Reorientation transition: tranverse bands (small $\varepsilon$) to longitudinal lanes (large $\varepsilon$).
- Flocking transition is a first-order liquid-gas phase transition.




S. Chatterjee, M. Mangeat, R. Paul, and H. Rieger, EPL **130**, 66001 (2020)
M. Mangeat, S. Chatterjee, R. Paul, and H. Rieger, Phys. Rev. E **102**, 042601 (2020)

## The q-state active clock model (ACM)

- Motivation: investigate the $q \to +\infty$ limit and see if the VM is recovered:
  - microphase separation in the coexistence phase,
  - absence of the reorientation transition,
  - quasi-long range order (QLRO) at $v \to 0$ limit.

S. Chatterjee, M. Mangeat, and H. Rieger, EPL **138**, 41001 (2022)

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- $N$ particles in a off-lattice $L_x \times L_y$ rectangular domain. Average density: $\rho_0 = N/L_xL_y$.
- $i$th particle characterized by position $\mathbf{x_i}$ and orientation $\theta_i \in \{0, 2\pi/q, \cdots, 2\pi(q-1)/q\}$.
- Neighborhood $\mathcal{N}_i = \{j \text{ with } |\mathbf{x_i} - \mathbf{x_j}| < 1\}$ of $i$th particle, constituted of $\rho_i$ particles.
- Local clock Hamiltonian and local magnetization in the neighborhood $\mathcal{N}_i$:

$$H_i^{\mathrm{ACM}} = -\frac{J}{2\rho_i} \sum_{k\in\mathcal{N}_i} \sum_{\substack{l\in\mathcal{N}_i \\ l\neq k}} \cos(\theta_l - \theta_k), \quad \mathbf{m_i} = \sum_{k\in\mathcal{N}_i} (\cos\theta_k, \sin\theta_k).$$

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- At each time $t$, the $i$th particle in state $\theta_i = \theta$ can
  - hop in preferred direction $\phi = \theta$ with a rate: $D(1+\varepsilon)$,
  - hop in another direction $\phi \neq \theta$ with a rate: $D[1 - \varepsilon/(q-1)]$,
  - flip to an orientation $\theta' \neq \theta$, for a temperature $T = \beta^{-1}$, with a rate:

$$W_{\mathrm{flip}}(\theta \to \theta') = \gamma \exp(-\beta \Delta H_i^{\mathrm{ACM}}) = \gamma \exp\left\{ \frac{\beta J}{\rho_i} \left[ \mathbf{m_i} \cdot (\mathbf{e}_{\theta'} - \mathbf{e}_\theta) + 1 - \cos(\theta' - \theta) \right] \right\}.$$

- Rescaled quantities (for an infinite $q$ limit well-defined):

$$\overline{D} = qD \qquad \text{and} \qquad \overline{\varepsilon} = \frac{\varepsilon}{q-1}.$$

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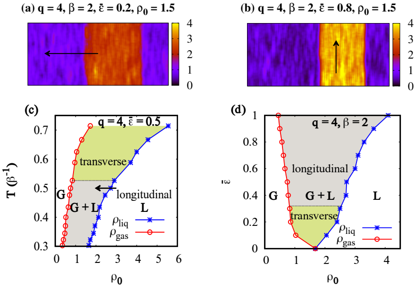
# Results for $q = 4$ and $q = \infty$

$q = 4$-state active clock model




- Liquid-gas phase transition, with macrophase separation in the coexistence region (like AIM and APM).
- Presence of the reorientation transition (like APM), from transverse band (low bias $\varepsilon$) to longitudinal lane (high bias $\varepsilon$).

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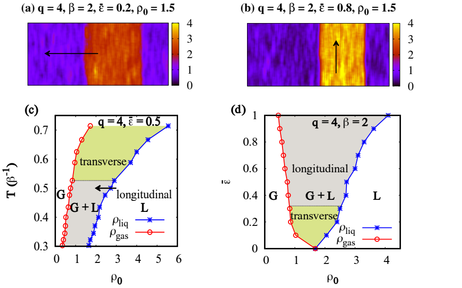


- Liquid-gas phase transition, with macrophase separation in the coexistence region (like AIM and APM).
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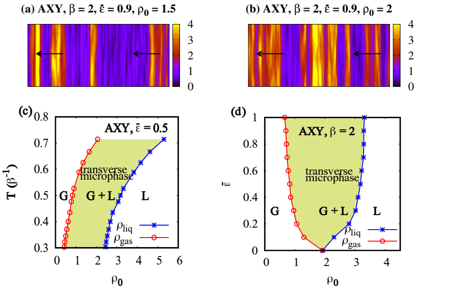
## active XY model ($q = \infty$)




- Liquid-gas phase transition, with microphase separation in the coexistence region (like VM).
- Absence of the reorientation transition (only transverse bands are observed).

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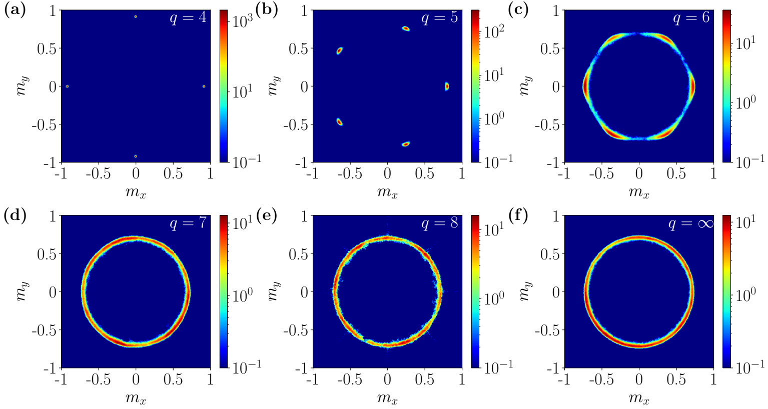
## $q$-dependence on the ordered phase without activity ($\varepsilon = 0$)

- Magnetization distribution for $\beta = 2$, $\rho_0 = 3$, in $50 \times 50$ domain.




- For $q \leq 5$: "pinned" orientations (LRO), and for $q \geq 6$: "unpinned" orientations (QLRO).
- The transition QLRO/LRO may depend on temperature, density and system size.

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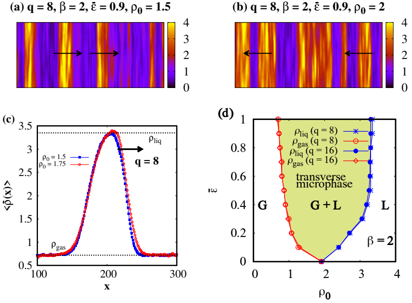
## Results for finite $q$ values

Coexistence phase and phase diagram

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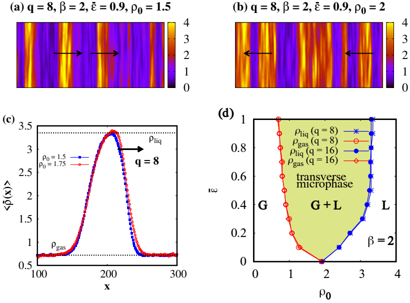
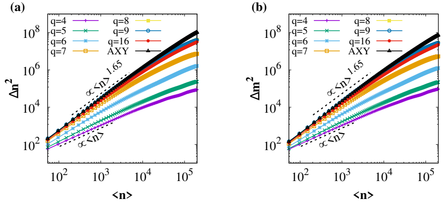



- Liquid-gas phase transition, with microphase separation in the coexistence region (like VM).
- Absence of the reorientation transition.

## Number and magnetization fluctuations in liquid phase




- $\Delta n^2 \sim \langle n \rangle^{\xi}$ and $\Delta m^2 \sim \langle n \rangle^{\xi}$, with

| $q$ | 4 | 5 | 6 | 7 | 8 | $\infty$ |
|---|---|---|---|---|---|---|
| $\xi$ | 1.04 | 1.08 | 1.36 | 1.56 | 1.62 | 1.65 |

- macrophase separation $\Rightarrow \xi = 1$, normal fluctuations ($q \leq 5$).
- microphase separation $\Rightarrow \xi > 1$, giant fluctuations ($q \geq 6$).
- No asymptotic regime at large $\langle n \rangle$ observed for our system sizes ($L_x = L_y \leq 800$).

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## Hydrodynamic description

- Hydrodynamic equations for the density $\rho(\mathbf{x};t)$ and the magnetization $\mathbf{m}(\mathbf{x};t)$:

$$\partial_t \rho = D_0 \nabla^2 \rho + \frac{v}{4} \nabla \cdot (\nabla \cdot Q) - v \nabla \cdot \mathbf{m},$$

$$\partial_t \mathbf{m} = D_0 \nabla^2 \mathbf{m} + \frac{v}{8} \begin{pmatrix} \partial_{xx} - \partial_{yy} & 2\partial_{xy} \\ 2\partial_{xy} & -\partial_{xx} + \partial_{yy} \end{pmatrix} \mathbf{m} - \frac{v}{2} (\nabla \rho + \nabla \cdot Q)$$
$$+ \gamma_0 \left[ \beta J - 1 - \frac{r}{\rho^{2-\xi}} - \kappa \frac{\mathbf{m}^2}{\rho^2} \right] \mathbf{m}.$$

with diffusion constant: $D_0 = \overline{D}/4$, self-propulsion velocity: $v = \overline{D}\overline{\varepsilon}$, ferromagnetic interaction strength $\gamma_0 = q\gamma/(q-1)$, $\kappa = (\beta J)^2 (7 - 3\beta J)/8$, and nematic tensor

$$Q = \frac{\beta J}{2\rho} \begin{pmatrix} m_x^2 - m_y^2 & 2m_x m_y \\ 2m_x m_y & -m_x^2 + m_y^2 \end{pmatrix}.$$

- Impossible to conclude about macrophase/microphase separation in the coexistence region with vectorial PDEs [Solon *et al.*, PRL **114**, 068101 (2015)]. We need to introduce a noise term (future work).

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# Conclusion

- For the $q = 4$-state ACM, we recover the same properties as the AIM and the APM: macrophase separation, presence of the reorientation transition, and LRO phase.
- For the AXYM ($q = \infty$), we recover the same properties as the VM: microphase separation, absence of the reorientation transition, and QLRO phase.
- For other $q$ values (at fixed $\beta = 2$ and $\bar{\varepsilon} = 0.9$, with $L \leq 800$): we observe macrophase separation for $q \leq 5$, and microphase separation for $q \geq 6$, as well as a QLRO/LRO transition at $\varepsilon = 0$.

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## Open questions and perspectives

- Do the number fluctuations in the liquid phase have an impact on the coexistence phase?
- Are the pinned/unpinned orientations in liquid phase equivalent to macrophase/microphase separation in coexistence region?
- Are the pinned/unpinned orientations equivalent to normal/giant number fluctuations in liquid phase?
- Many differences observed compared to the study made by Solon *et al.* [PRL **128**, 208004 (2022)], for a slightly different model (different hopping/flipping rates). From where these differences arise?
- Work on the hydrodynamic description to validate the simulation's results.

# Thank you for your attention !

Following presentations

- Tuesday 28 at 12:00 PM: S. Chatterjee *et al.*, Flocking of unfriendly species: The two-species Vicsek model (BP 9.10 / TOE 317).
- Wednesday 29 at 03:45 PM: T. Guérin *et al.*, How stickiness can speed up diffusion in confined systems (DY 32.4 / ZEU 160).
- Thursday 30 at 01:00 PM: M. Mangeat *et al.*, Wetting of reflecting plates by an active Brownian fluid (DY 42.10 / P1).

References


- S. Chatterjee, M. Mangeat, R. Paul, and H. Rieger, *Flocking and reorientation transition in the 4-state active Potts model*, EPL **130**, 66001 (2020).
- M. Mangeat, S. Chatterjee, R. Paul, and H. Rieger, *Flocking with a q-fold discrete symmetry: Band-to-lane transition in the active Potts model*, Phys. Rev. E **102**, 042601 (2020).
- S. Chatterjee, M. Mangeat, and H. Rieger, *Polar flocks with discretized directions: the active clock model approaching the Vicsek model*, EPL **138**, 41001 (2022).